



Original Research Paper

Structure-Preserving Fractal Image Generation Using the Proposed Image Morphology-Based Retargeting Algorithm

Arjun Rai Dutta¹, Surbhi Saroha², Ankit Garg³

¹Research Scholar, Shobhit University, Meerut, Uttar Pradesh, India
Email: araidutta@gmail.com

²School of CSE, Shobhit University, Meerut, Uttar Pradesh, India
Email: surbhisaroha12@gmail.com

³UCRD, UIE, Chandigarh University, Mohali-140413, Punjab, India
Email: ankitgitm@gmail.com

Abstract: To construct fractal images, affine transformations are iteratively applied to an initial set of images. In the existing affine transformation framework, a scaling operator is generally used to change the aspect ratio of an image. However, the incorporation of the scaling operator may produce structural deformation in salient regions of the image. As a result, prominent reflective regions may become distorted as the number of iterations during fractal image generation increases significantly. To minimize deformation in reflective regions, this paper proposes an efficient affine transformation that incorporates a Proposed Image Morphology-Based Retargeting Algorithm (PIMRA) to replace the conventional scaling operator. To achieve this objective, the entire system is divided into two models. In the first model, algorithms for translation, rotation, and shearing are proposed. To modify the image dimensions in a content-aware manner, PIMRA is utilized. To evaluate the effectiveness of all the proposed algorithms, parameters such as luminance(l), contrast(c), and structure(s) are considered. Objective Image Quality Assessment (OIQA) is performed using the Structural Similarity Index Measure (SSIM). The IQA results show that the performance of PIMRA is 3.25% higher than that of the existing scaling operator. In the second system model, all the proposed geometric transformation techniques are combined to develop an affine transformation framework. The obtained values from IQA showcase that the performance of PAT is 10.45% and 12.45% higher than TAT when the images are selected from the RetargetMe and SIR² dataset respectively.

Keywords: Image retargeting; fractal; iterated function system; affine transformation; scaling; seam carving.

1. Introduction

The fractals are complex patterns that have the property of self-similarity. The most common forms of fractals are the structure of trees, rivers, coastlines, mountains, clouds, seashells, hurricanes, etc. Fractals are infinitely complex patterns that are self-similar across different scales. Self-similar fractals can be created by the most widely known random and deterministic algorithms [1-4]. To generate fractal images, three steps can be followed: (a) development of a set of required geometric transformations, (b) development of an initial pattern, (c) apply composite geometric transformations on the initial pattern, (d) repeat step (c) iteratively in an ongoing feedback loop [5]. Fractal geometry plays a significant role in various widely known application areas such as image compression [6], image encryption [7], graphics and design [8], fractal antennas [9-10], modern manufacturing [11], architecture design [12], and medical science [13]. In the field of fractal geometry,

fractal images are generally constructed by iteratively applying an affine transformation that is contractive in nature. The scaling operator utilized in the affine transformation can change the dimensions of an image either uniformly or non-uniformly. From the literature [14], it has been found that the use of a scaling operator can produce deformation in the salient objects of an image. When the number of iterations becomes very high, these distortions can be significantly perceived by the human visual system. To suppress distortion in the regions of interest, the integration of advanced image retargeting techniques is therefore important. To address this issue, this paper proposes a system model that is mainly divided into two different models. In the first model, different geometric transformation techniques are proposed and evaluated based on the selected parameters, while in the second model, all the proposed techniques are combined to generate visually appealing fractal images with a significantly reduced percentage of deformation. In both system models,

*Corresponding Author:

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objective image quality assessment (OIQA) is conducted using the structural similarity index metric (SSIM) [15]. To evaluate the performance of the proposed affine transformation (PAT), a total of 20 iterations are applied to generate the final attractor. Since the size of the attractor becomes considerably reduced after repeated iterations, a zoom operation is applied to a selected region of the final attractor to facilitate visual inspection. This enables the structural deformation and visual quality of the generated fractal image to be examined more clearly by the human visual system. To conduct all the experiments, an SIR² data set is used, which contains images having different attributes such as line structure, curves, symmetry, reflective regions, and foreground & background regions.

In this paper, Section 2 provides the literature review that mainly focuses on the research work and studies related to fractal image generation and content-aware image retargeting techniques. Section 3 presents the proposed methodology of the research work that explains two different system models. Further, the mathematical model of both the systems is constructed. Section 4 presents the working and experimentation of the proposed models. In this section, the proposed geometric transformation algorithms are discussed in detail. Section 5 focuses on the dataset that is used to select different images to generate the fractal images. Section 6 presents results and discussion based on the two experiments carried out to justify the performance of the PIMRA and PAT. To assess the obtained results, OIQA is performed. In the end, the paper is concluded with some future directions.

2.1 Fractal Image Generation

In this section, the relevant papers are selected and reviewed that are related to fractal image generation to observe the process of the iterated function system (IFS). In the presented literature review, it is observed that different types of fractal images can be generated using the affine transformation function. The conventional affine transformation applies a set of contractive geometric transformations over the image iteratively under an iterated function system. In the TAT, downsampling of the input image is usually carried out by a scaling operator. Nurujjaman et al. [16] in their study conducted experimentation to understand the different properties of fractals. The main objective of the study is to pay attention to different mathematical behavior of fractal patterns to develop new formulas related to fractals. Sallow et al. [17] proposed an efficient algorithm to generate Mandelbrot and Julia sets using both sequential and parallel implementations. The central processing unit (CPU) and graphical processing unit (GPU) are used for sequential and parallel implementation,

respectively. Zhu et al. [2] presented the applications of IFS in the construction of Sierpinski fractals based on the Matlab environment. In the study, it is found that the deterministic algorithm can be used to obtain the aesthetically pleasing structure of a 2D general Sierpinski fractal. Indrani et al. [18] proposed an efficient fractal image compression technique. The algorithm requires less computational time to encode and decode the image as compared to fractal encoding. Guedri et al. [19] presented a 3D model of a tree of retinal blood vessels. In the first stage of the proposed model, a 3D thinning algorithm is utilized to identify the blood vessel centerline. Further, in the second stage, a widely known Douglas–Peucker 3D technique is applied to compute the characteristic points. Rama et al. [20] presented the significant role of iterated function systems in the generation of beautiful fractal patterns. In the study, the number of iterations and time is computed to generate various types of fractal patterns under IFS. Hadzieva et al. [21] introduced an efficient tool that is capable of transforming 2D fractal images. To generate a fractal image, barycentric coordinates are assigned to the points. The introduced tool utilizes the prominent properties of the barycentric coordinates and affine transformation. Garg et al. [22] analyzed the usefulness of linear mapping that is applied between the attractor and source image. The analysis presents that the effective linear mapping of image pixels under affine transformation preserves the three main properties of the line structures, such as parallelism, collinearity, and orientation. Tao et al. [23] investigated the impact of IFS code in the generation of various fractal images. In the investigation, different types of beautiful fractal images are generated with the variation of controllability parameters under affine transformation. Rama et al. [24] demonstrate the generation of two popular fractal images, such as the Mandelbrot Set and the Julia Set. Younes et al. [25] in their study presents a detailed description of fractal patterns and their mathematical modeling using different fractal generation methods. The study presents different types of software to generate beautiful fractal patterns. Garg et al. [26] explore various methods and theorems of fractal image generation under an iterated function system. The investigation reveals the usefulness of the escape time algorithm that is mainly used to visualize the dynamics related to the Julia sets and Mandelbrot sets. Kumar et al. [27] proposed an efficient framework that mainly focuses on the generation of fractal objects. The proposed framework presents a mathematical-based approach that is leveraged by web technology. The users can utilize the proposed framework to generate various fractal patterns specifically for ornaments based on some predefined inputs. Belousova et al. [28] proposed some methods

to generate aesthetically pleasing fractal patterns. To generate fractal patterns, OpenGL Shading Language is used, which leverages the performance benefits of the GPU. Murodillaevich et al. [29] proposed an algorithm to generate fractal-shaped objects. In the proposed algorithm, various analytical equations are used to model beautiful objects with fractal structures. The novelty of the proposed algorithm is the utilization of the R-function, the binomial theorem, and 2D geometric transformation in its development.

2.2 Content-Aware SC

In this section, an intensive literature review is carried out to identify a suitable image retargeting algorithm that is based on the conventional SC technique. Avidan and Shamir [30] proposed an SC image retargeting technique to alter the aspect ratio of the image. The novelty of the proposed algorithm is the elimination of least energy pixels from the image that are not highly noticeable to the human eyes. In the literature [31], the limitations, improvements, and possible solutions to the SC technique are intensively analyzed based on OIQA and SIQA. Achanta et al. [32] suggested an improved image resizing technique based on a novel saliency detection scheme. In the proposed method, saliency maps are used to identify important regions of the image by measuring the global saliency of pixels, which considers intensity and color features. Goferman et al. [33] suggested context-aware saliency. The usefulness of the suggested method is observed in two applications where the context of the prominent objects is as important as the objects themselves. To preserve the structure, Mishiba et al. [34] proposed seam merging and a new merging energy criterion. Due to the lack of use of pixel importance, the method could not safeguard salient contents. Hence, to enhance seam merging, Mishiba et al. [35] used importance and structure energies. In the experimentation, the structure energy of the original image is estimated to safeguard the main structures by taking a cartoon version of the source image. A new energy term has been introduced to overwhelm the distortion produced by extreme shrinking or enlargement at the time of iterative merging or inserting (Mishiba et al. [36]). Wang et al. [37] proposed a video seam-carving algorithm. In the proposed algorithm, the structure of important objects is preserved. Curves of source and retargeted video frames are compared, and the additional matching cost is computed. Chen et al. [38] proposed a method through which an energy map can be obtained by combining the traditional $L-1$ norm of the gradient with depth-aware saliency (3D saliency). Choi et al. [39] proposed a modified efficient seam-carving algorithm to preserve the salient part of the image and also maintain the important structures of the image which are easily visible to the human eye. Lin

et al. [40] proposed a saliency-based seam-carving algorithm. According to the greyscale of saliency detection, images are classified. The foreground and detailed area of a subject and edge can be protected by adding various protection methods. Li et al. [41] proposed a method that utilizes suboptimal and discontinuous seams for SC. Spatio-temporal coherence can be preserved by allowing optimal seams to travel in the homogeneous regions of the image. Patel et al. [42] proposed a method in which a multiple pixel wide seam is inserted and removed in a single iteration to accelerate the process of SC. To minimize the effect of the false edge, the energy of pixels that are adjacent to the pixels included in the multiple pixel seam is also minimized. Garg et al. [43] proposed an advanced SC technique that effectively merges the disjoint image segments. To merge the segments, the proposed image stitching method is used. During the merge of segments, the technique overcomes the challenge of false seam generation.

3. Proposed Methodology

The methodology of the research work is primarily focuses on the development of a system architecture that is required for the development of PAT. Another objective of the system architecture is to minimize the loss of color and structural information while generating the fractal images. To accomplish this task, the entire system architecture is divided into two different parts. The first part i.e. system model-1 mainly focuses on the development of individual geometric transformation algorithms such as translation, rotation, and shearing. The second part, i.e., system model-2, tries to combine proposed geometric transformation algorithms to develop a PAT. The system model-1 checks the efficiency of proposed geometric transformation algorithms based on OIQA. Moreover, the system model-2 evaluate the efficiency of PAT based on OIQA. The details of the entire system architecture are given below.

3.1 Proposed System Models

The entire architecture of the system model focuses on the development of an efficient PAT. For its effective development, a number of operations are being performed, which are listed below.

- Development of geometric transformation algorithms
- Comparison of proposed geometric transformation algorithms with existing transformation algorithms
- Development and selection of efficient composite transformation sequence
- Development of efficient affine transformation function
- Evaluation of system outcomes based on the OIQA

- Analysis of system outcomes based on the parameters

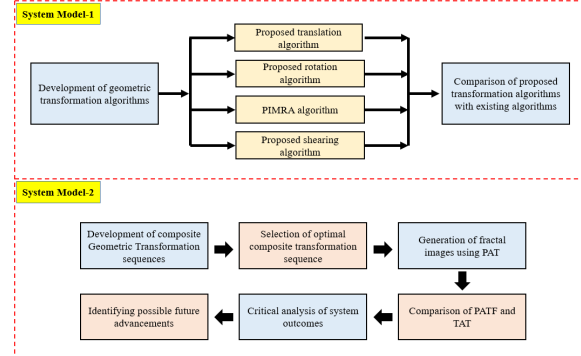


Fig. 1. Architecture of proposed System Models

Fig. 1 shows the architecture of the system model for the effective development of PAT. The details of the mathematical modeling of the entire system architecture are given in subsequent sections. After the design of the proposed system architecture, both of its subsystems are further considered for their mathematical modeling.

3.1.1 Mathematical Modeling: System model-1

In this section, the mathematical modeling of each proposed geometric transformation algorithm is discussed in detail. The mathematical model of each proposed geometric transformation algorithm is further used in System Model-2 for the construction of PAT. In addition to this, the modeling of the linear interpolation method is also explained to obtain the relative positions after performing each geometric transformations. In the proposed architecture, the goal of linear interpolation is to estimate the exact color components for each transformed pixel.

(a) Development of Translation

The mathematical formula for translation transformation is used to shift the image pixel in the transformed image. To accomplish this task, the new translation factors t_x and t_y are supplied by the user. To translate the image pixels in the target image, both the translation factors are added iteratively with the corresponding x and y coordinates of the source image. To add the translation factors in both horizontal and vertical directions eq. (1) is used. To obtain the color information for each transformed pixel, a linear interpolation is applied to the image. After obtaining the interpolated image, a reshape operation is applied to convert the interpolated image into a similar source image having the same dimension. The mathematical model for translation is tested based on different translation factors such as $t_x = t_y = 0$, $t_x = t_y > 0$, $t_x = t_y < 0$, $t_x \neq t_y > 0$, $t_x \neq t_y < 0$.

$$v = \begin{bmatrix} x \\ y \end{bmatrix}; v' = \begin{bmatrix} x' \\ y' \end{bmatrix}; t = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$v' = v + t \quad (1)$$

where $x' = x + t_x$ and $y' = y + t_y$

(b) Development of Rotation

The mathematical model for rotation transformation is constructed to change the orientation of the source image. To generate the fractal images, the source image can be rotated both in clock-wise and anti-clockwise directions.

To generate the rotation matrix in both clockwise and anti-clockwise directions eq. 2 & 5 are used, respectively.

$$x' = r \cos(\theta + \phi) = r \cos \phi \cos \theta - r \sin \phi \sin \theta \quad (2)$$

$$y' = r \sin(\theta + \phi) = r \cos \phi \sin \theta + r \sin \phi \cos \theta \quad (3)$$

In eq. 2 & 3, the values of $x = r \cos \phi$ & $y = r \sin \phi$ are substituted to get two new equations, i.e., eq. (4 & 5). After substitution, the rotation matrices are obtained for both clockwise and anticlockwise directions.

$$x' = x \cos \theta - y \sin \theta \quad (4)$$

$$y' = x \sin \theta + y \cos \theta \quad (5)$$

To obtain the exact color component in the target image linear interpolation is applied.

(c) Existing PIMRA

The existing PIMRA is mainly used to replace the scaling operator in the PAT. The advantage of incorporating PIMRA is to suppress the inclusion of deformations when the color similarity between foreground and background regions is almost identical. To compress and extend the dimension of the image, two operations are performed by the PIMRA algorithm, which are (a) Computation of base energy map of the source image using gradient method, (b) Energy augmentation operations using morphological operations on the prominent object boundaries.

- PIMRA Phase-1: The primary goal of PIMRA [44] is to detect those pixels that separate foreground and background ROIs where the color composition is almost the same. Using the Canny edge detection operator, PIMRA first recognizes the noticeable edges. The edge map is then used to calculate the intensity of all pixels that are on the edges of objects. After determining the positions of all edge pixels, the maximum and minimum values are calculated and saved in an image matrix. The image matrix is then subjected to a Gaussian filter using a 2-D Gaussian smoothing kernel with a standard deviation. The filtered image is then utilized to calculate object perimeter pixels. If the filtered image pixels have a nonzero value and are linked to at least one zero-valued pixel, they are regarded as part of the perimeter. An 8-way 2-D connection is used to identify two neighboring pixels that can be regarded as

part of the same object area. The PIMRA can quickly discover image pixel connections in horizontal, vertical, or diagonal orientations. After that, a binary image is formed by replacing all values greater than a globally selected threshold with 1s and changing all other values to 0s. Otsu's approach is used to determine the threshold value to minimize the interclass variance of the thresholded black and white pixels. The threshold is calculated during the PIMRA binarization process to assign the class of the image pixels. Image pixels with intensities larger than the set threshold can be regarded as the foreground areas, while others can be regarded as the background regions. The included Otsu's technique is used to minimize intensity fluctuation within the same cluster while increasing inter-cluster variance. After detecting the image clusters, the dilation image morphological operation is conducted on the boundaries of objects to expand their width. The PIMRA dilation method adds pixels to the conspicuous boundaries of objects. The number of pixels added to the object border is determined by the size of the structuring element used. To perform the morphological dilation procedure, a flat structuring element with a radius of 5 units is chosen to pick the true-valued pixels and ignore the false pixels.

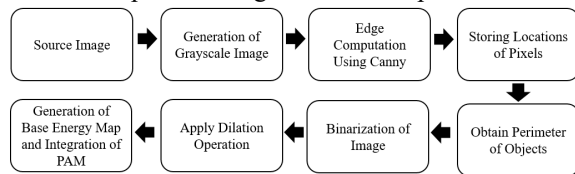


Fig. 2. Construction of base energy map

The key rationale for using a flat structuring element is that it can be used with both binary and grayscale images. A color map of the morphological image is formed once the pixels at the object boundaries are added. After the dilation operation, the energy of pixels encountered at the border of the objects is enhanced to generate the base energy map. The major rationale for the energy increase is to limit the occurrence of ideal seams converging on the boundary of the prominent features. The base energy map is further combined with PAM to identify reflective sensitive regions and to route the ideal seams to neighboring areas to preserve the loss of information. Fig. 2 depicts the steps carried out to construct the base energy map. The details of significant steps of PIMRA are given below.

- **Generation of edge map using canny operator:** In the first step of the PIMRA, source images are chosen to detect the notable edges. A Canny edge detection operator is used to recognize a wide variety of edges by considering numerous steps. The Canny operator performs five main operations, namely: (a) noise reduction, (b) computation of intensity variation,

(c) edge thinning, (d) identification of strong, weak, and non-relevant edge pixels, (e) edge identification.

a) **Noise reduction:** The fluctuation in pixel intensity is calculated in image processing using gradient maps. The edge detection findings might be quite sensitive to the noise in the image. To smooth the conspicuous edges, a convolution process using Gaussian blur can be employed. The kernel size is effectively determined in order to impart the blurring effect to the image. The Canny method employs a 5×5 Gaussian kernel to get the desired blurring effect. Eq. (6) can be used to build a 5×5 Gaussian filter kernel.

$$H_{ij} = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(i-(x+1))^2 + (j-(x+1))^2}{2\sigma^2}\right); 1 \leq i, j \leq (2x+1) \quad (6)$$

In eq. (6), σ shows the standard deviation of the distribution in the x and y directions.

b) **Image gradient computation:** The Canny algorithm determines the strength and direction of the conspicuous edges. As a result, the gradient is calculated after edge detection operators are applied. To achieve the directional change in intensity, a filter can be applied in both horizontal and vertical directions. A 3×3 Sobel filter may be used to extract the derivative in both directions in order to create the gradient map. Eqs. (7 & 8) are used to calculate the gradient magnitude and the directional change in intensity. After acquiring the edges, it is discovered that some are thick and others are thin, owing to differences in intensity. As a result, the Canny algorithm employs non-maximum suppression over the edges to counteract this effect.

$$|Gd| = \sqrt{Img_x^2 + Img_y^2} \quad (7)$$

In eq. (7), the variables Img_x^2 & Img_y^2 reflect the fluctuation in intensity in x and y directions, respectively. The gradient map of the source image is denoted by the variable Gd .

$$\alpha(x, y) = \tan^{-1}\left(\frac{Img_x}{Img_y}\right) \quad (8)$$

In eq. (8), the variables Img_x and Img_y reflect the change of gradient in horizontal and vertical directions respectively, $\alpha(x, y)$ represents the direction of intensity fluctuation.

c) **Non-Max Suppression:** This process is used to thin detected edges. In the Canny algorithm, gradient intensity is analyzed and only pixels with the highest intensity are kept as edges, while others are suppressed. Since this process makes edge intensity inconsistent, double thresholding and hysteresis edge tracking are applied to obtain accurate and continuous edges.

d) Double threshold: The main objective of this process is to classify edge pixels into three types: strong, weak, and non-edge (quasi) pixels. This classification is based on pixel intensity and threshold values. Strong pixels have intensity above the higher threshold, weak pixels lie between the two thresholds, and quasi (non-edge) pixels have intensity below the lower threshold. To find strong edges, weak edges, and non-relevant edges eq. (9-11) are used respectively.

$$Int > HT$$

(9)

$$LT < Int \leq HT$$

(10)

$$Int \leq LT$$

(11)

In eq. (9-11) Int , LT , HT present intensity, low threshold, and high threshold, respectively.

e) Edge Tracking by Hysteresis: The method is used to convert weak edge pixels into strong edge pixels. To perform this modification, at least one of the strong edge pixels is computed in the local neighborhood of the weak pixels. If high intensity edge pixels are identified in its proximity, the low intensity pixel is changed.

- Calculation of object perimeters: In the PIMRA, the major goal of this phase is to compute the perimeter pixels in the binary image. After computation, a binary image consisting of the pixels on the image objects' perimeters is produced. In the computation, the pixels are considered as a part of the object perimeter based on two criteria: (a) the value of the pixel is non-zero, and (b) the pixel is adjacent to at least one zero-value pixel. To identify pixels with non-zero values, Eq. (12) is used.

$$Int(x, y) \neq 0$$

(12)

where Int is the intensity of pixels at location x and y . Further, to identify pixels adjacent to at least one zero-value pixel eq. (13) is used.

$$\exists(i, j) \in K(x, y): Int(i, j) = 0$$

(13)

where $K(x, y)$ presents the set of 4-connect neighbourhoods of pixel at location (x, y) .

Therefore, based on eq. (12) and eq. (13), the condition of perimeter pixels can be represented in the form of eq. (14).

$$\begin{cases} 1 & \text{if } Int(x, y) \neq 0 \text{ and } \exists(i, j) \in K(x, y): Int(i, j) = 0 \\ 0 & \text{otherwise} \end{cases}$$

(14)

A default 4-pixel connectivity is used since the approach processes 2D images. The connectivity parameter ensures accurate extraction of object boundaries. However, issues arise when inner object boundaries (holes) become disconnected. To address this, a hole-filling process is applied, followed by an

expansion (dilation) operation to eliminate holes within the objects.

- Binarization of edge map: The main goal of edge binarization is to determine an optimal threshold and separate pixels into foreground and background. Pixel intensities are compared with this threshold—values above it are assigned to the foreground, while those below it belong to the background. A global threshold is applied to convert the image into a binary form (0s and 1s). Otsu's method is used to find the optimal threshold by minimizing the intra-class variance of the two pixel groups. Eq. (15) gives the variation inside the class at any threshold t .

$$\sigma^2(t) = w_{back}(t)\sigma_{back}^2(t) + w_{fore}(t)\sigma_{fore}^2(t) \quad (15)$$

where $w_{back}(t)$ and $w_{fore}(t)$ reflect the probability of the count of pixels associated to each class at threshold t and σ^2 signifies color value variance.

To define the weights eq. (16 & 17) is used.

$$w_{back}(t) = \frac{P_{back}(t)}{P_{total}}$$

(16)

$$w_{fore}(t) = \frac{P_{fore}(t)}{P_{total}}$$

(17)

where P_{back} represents the count of background pixels and P_{fore} signifies the count of foreground pixels respectively at threshold t . P_{total} shows the total number of pixels in the image.

To compute the variance, eq. (18) is used.

$$\sigma^2(t) = \frac{\sum(a_i - \bar{a})^2}{K-1} \quad (18)$$

where, a_i is the value of the pixel at i in the cluster (background or foreground), $\bar{a}$ is the average of pixel values in the clusters (background and foreground), and K is the number of image pixels.

- Dilation image morphological operation: The primary goal of this stage is to execute the dilation morphological operation in order to add pixels to the boundaries of the objects. Let I be the binary image and S is the structuring element. The dilation operation of I by S is defined in eq. (19)

$$I \oplus S = \{z | (B)_z \cap I \neq \emptyset\} \quad (19)$$

where $(B)_z$ is the translation of B by z , and the dilation $I \oplus S$ includes all points z such that the structuring element S translated by z intersects with I .

The PIMRA generates the seams that may be directed towards the objects' boundaries. The junction of the ideal seams on the outlines of the item might alter their shape. As a result, it is strongly advised to divert the ideal seams to a nearby place prior to the occurrence of ideal seam junction. The process of seam distraction can be optimized after adding pixels to the object border and raising the energy of the boundary pixels.

The energy map E can be defined using eq. (20) in terms of the gradient magnitude.

$$E(x, y) = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2} \quad (20)$$

where ∂I showcase the gradient of the image I and $E(x, y)$ shows the energy of the pixel at location (x, y) .

The goal is to guide optimal seams toward low-energy pixels while avoiding object boundaries, as seams crossing borders can cause visible distortion. To achieve this, a structuring element based on the shape and size of objects is used to define the neighborhood. A flat structuring element helps detect true boundary pixels. The base energy map is then adjusted to redirect seams away from object edges by increasing the energy of boundary pixels, ensuring seams pass through less noticeable regions.

- Finding and saving the edge pixels location: After acquiring the dilated edge map, the positions of all edge pixels are calculated. After computing all the locations, the edge pixel's local minimum is determined using eq. (21).

$$L_Min(i) = \min \{D_d(i, j) \mid j = 1, 2, \dots, n\} \quad (21)$$

where L_Min shows the local minimum, and D_d presents the dilated edge map.

Similarly, to obtain the local maximum of edge pixels eq. (22) is used.

$$L_Max(i) = \min \{D_d(i, j) \mid j = 1, 2, \dots, n\} \quad (22)$$

where L_Max shows the local minimum, D_d presents the dilated edge map, x is the row index, and y is the column index.

The local minimum and maximum edge positions of each row are stored in a 2D array. After collecting all edge locations, a Gaussian blur is applied to smooth the edges, with different standard deviation values tested to control the blur. A standard deviation of 1 is selected as optimal. For computational efficiency, the Gaussian filter size is rounded to the nearest odd integer. Gaussian kernel coefficients are sampled from the 2D Gaussian function given in eq. (23).

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \quad (23)$$

In eq. (23), σ represents the standard deviation of the Gaussian distribution, and (x, y) are the coordinates in the image kernel.

Two 1-D Gaussian functions are multiplied together to generate the 2-D Gaussian function for the horizontal and vertical direction.

f) PIMRA Phase-2.2: In PIMRA phase 2.1, the PAM [44] is combined with the base energy map to serve two purposes: (a) suppress distortions in sensitive reflective regions, and (b) preserve

background and foreground regions where color composition is almost similar. To efficiently identify ROIs, the base energy map and reflection-aware mask are concatenated using a matrix multiplication. The PAM recognizes the notable edges, intensity fluctuation, and salient objects in ROIs. The fundamental goal of the PIMRA is to inhibit structural deformation on the object's border. As a result, the ideal seams are steered to surrounding portions of the picture to achieve this goal. Using dynamic programming, the PIMRA determines the best seam routes. The base energy map and reflection-aware mask in PAM offer border location of objects and sensitive reflective regions, respectively. The obtained data is saved in the 2D array for further processing. The width of the edges in the PAM becomes enlarged using the dilation image morphological technique. Eq. (24) is used to apply the dilation operation on the selected images. According to Eq. (24), the output element Z has at least one element in S that intersects with the element accessible in the reference image I .

$$I \oplus S = \{z \mid [(S)_z \cap I] \in I\} \quad (24)$$

where I represents the original image, S represents the structuring element, and Z denotes the output element. After dilation, border pixel positions are stored in a second 2D array. During seam generation, the lowest energy pixels are identified, and PIMRA compares their positions with border pixels. If a match occurs, seams are redirected to nearby regions by increasing the energy of boundary pixels, preventing them from crossing object edges. This ensures seams pass through less important areas. Once all seams are identified, boundary energies are restored to normal. This process is repeated in each iteration to achieve content-aware retargeting. However, PIMRA has higher complexity than standard seam carving due to seam redirection, morphological operations, and increased computational cost. The aforementioned factors contribute to image retargeting outcomes that are more aesthetically appealing than SC. The impact of the dilation image morphological operation can be inverted after applying the erosion operation at the boundary of the objects. Eq. (25) is used to perform the erosion image morphological procedure.

$$I \ominus S = \{z \mid (S)_z \in I\} \quad (25)$$

In Eq. (25), S and I represent structural element and the original image, respectively. When the structuring element is a portion or equivalent to the binary image I , the resultant element Z is examined accordingly.

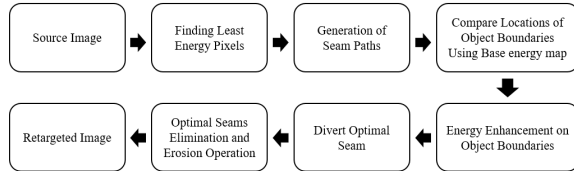


Fig. 3. Process of the PIMRA in conjunction with PAM and base energy map

g) Shearing: Shearing transformation is applied to change the shape of the image in both horizontal and vertical directions. For this purpose, two shearing factors such as sh_x and sh_y are computed. The generalized form of shearing transformation in both directions is given in eq. 26 & 27. In eq. (26 & 27), x and y are the locations of pixels in the original image, while x' and y' are the corresponding locations of pixels in the sheared image.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ sh_x & 1 \end{bmatrix} \quad (26)$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \times \begin{bmatrix} 1 & sh_y \\ 0 & 1 \end{bmatrix} \quad (27)$$

h) Linear Interpolation: Linear interpolation is applied to estimate the approximated value of the new pixel after applying a transformation. Suppose p, q, r and s are the positions of the pixels for which the computation of the color component is required. Firstly, the relations among P, p , and Q are computed using eq. 28. To obtain eq. 30 eq. 28 & 29 are used.

$$\frac{p-P}{\alpha} = \frac{Q-P}{\alpha} \quad (28)$$

$$p = P + \frac{\alpha(Q-P)}{\alpha} \quad (29)$$

$$p = P + \alpha(Q - P) \quad (30)$$

Similarly, after computing the relationship among R, r , and S the color components for r are obtained by eq. 31.

$$r = R + \alpha(S - R) \quad (31)$$

After obtaining eq. 31, the relationship among p, Q , and r is computed to obtain the color components for q using eq. 32.

$$q = p + \beta(r - p) \quad (32)$$

To obtain the color components of all the image pixels, eq. 33 is used. This equation is obtained by substituting eq. 30 & 31 in eq. 32.

$$q = P + \alpha(Q - P) + \beta(R + \alpha(S - R) - (P + \alpha(Q - P))) \quad (33)$$

3.1.2 Mathematical Modeling: System Model-2

This section combines all the proposed geometric transformation algorithms along with PIMRA to construct PAT. To obtain a better sequence of transformations, several sequences of composite

transformations are tested. The tested sequences of composite transformations are shown in Fig. 4.

$$\begin{aligned} CT_1 &\rightarrow [O] \times [T] \times [D] \\ CT_2 &\rightarrow [O] \times [T] \times [S_h] \\ CT_3 &\rightarrow [O] \times [T] \times [R] \\ CT_4 &\rightarrow [O] \times [D] \times [R] \\ CT_5 &\rightarrow [O] \times [D] \times [S_h] \\ CT_6 &\rightarrow [O] \times [R] \times [S_h] \\ CT_7 &\rightarrow [O] \times [D] \times [R] \times [S_h] \\ CT_8 &\rightarrow [O] \times [T] \times [R] \times [S_h] \\ CT_9 &\rightarrow [O] \times [T] \times [D] \times [S_h] \\ CT_{10} &\rightarrow [O] \times [T] \times [D] \times [R] \\ CT_{11} &\rightarrow [O] \times [T] \times [D] \times [R] \times [S_h] \end{aligned}$$

Fig. 4 Different composite geometric transformations; [CT]: Composite Transformation, [O]: Original image, [T]: Translation, [R]: Rotation, [D]: Downsampling, [Shx]: Shearing

The obtaining the efficient composite transformation, the mathematical modeling related to PAT is developed, which is discussed below.

An affine transform can be obtained by applying a linear transformation followed by a translation. A linear transformation from P^n to P^m is a mapping Z that satisfies eq. 34.

$$Z(\alpha x + \beta y) = \alpha Z(x) + \beta Z(y) \quad (34)$$

For all $x, y \in P^m$ and all scalars α, β .

Let Z be a linear transformation from P^n to P^m . Then there is an $m \times n$ metric L such that

$$Z(x) = Lx, \quad x \in P^n \quad (35)$$

One significant property of linear transformations is that they map line segments to line segments. This is modeled by considering the vector-valued function, which is given in eq. 36.

$$s(t) = ty + (1 - t)x, \quad 0 \leq t \leq 1 \quad (36)$$

The vector-valued function traces the line segment from x to y . Let Z be a linear transformation. After defining the property of a linear transformation, eq. (37) is obtained.

$$Z(ty + (1 - t)x) = t Z(y) + (1 - t) Z(x) \quad (37)$$

A translation of P^n is a mapping of the form given in eq. (38).

$$T(x) = x + a, \quad x \in P^n \quad (38)$$

In eq. (38), a is a fixed or constant vector.

By combining a linear transformation with a translation, the affine transformation can be represented in matrix-vector form as given in eq. (39)

$$T(x) = Ax + a, \quad x \in P^n \quad (39)$$

In the case of P^n it becomes a function of affine transformation which is given in eq. (40)

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} \quad (40)$$

4. Working and Experimentation of Proposed Models

After mathematical modeling of system (1 & 2), two experimentations are being performed in each system model. The main objectives of both experiments are to apply, compare, and assess the proposed geometric transformation algorithms.

4.1 Working of proposed system model-1

The experimentation (phase-1) is mainly concerned with the applicability and evaluation of the proposed geometric transformation algorithms and similar traditional algorithms. The flowchart of experimentation (phase-1) is given in Fig. 5.

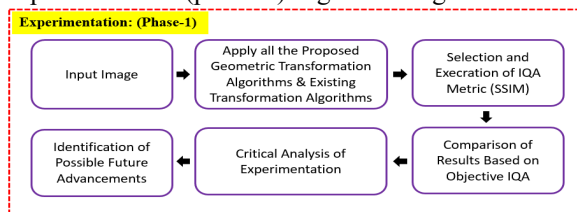


Fig. 5. Flow of experimentation (Phase-1)

In experiment phase-1, the objective of the selected parameters is to assess the performance of the PIMRA and other proposed geometric transformation techniques. Variations in parameter values are closely observed through OIQA. To obtain parameter values, SSIM is used, which generates different local and global SSIM maps. In this phase, the performance of conventional scaling operator is also compared with PIMRA. After comparison, it is analyzed that incorporation of PIMRA in the PAT produces better results as compared to scaling. In addition to this, it is also observed that the visual quality of the results obtained from PIMRA is highly preserved as compared to the scaling operator. In the experiment (phase-1), firstly an input image having a high resolution is selected. After that, each proposed transformation algorithm is applied to the different types of source images to justify their efficiency. Firstly, the efficiency of the proposed translation algorithm is tested based on various types of translation factors. The efficacy of the proposed algorithms are assessed based on translation factor, scaling factors, and shearing factors in positive and negative directions. Secondly, the PIMRA is tested by iteratively changing the dimension of the input images. The dimension of the input images is changed by artificially inserting optimal seam both in horizontal

and vertical directions. The PIMRA also performs its major operations, i.e., energy augmentation and seam diversion, iteratively to minimize the effect of distortion on the salient region of the image. Thirdly, the effect of the proposed rotation transformation algorithm is tested by rotating the input in both clockwise and anticlockwise directions. Finally, based on the different shearing factors, the efficacy of the proposed shearing transformation is analyzed. In experiment phase-1, the efficacy of TAT is also analyzed with the same factors of geometric transformations.

4.2 Working and experiments of system model-2

This section presents the steps that are followed by system model-2 to conduct the experimentation (phase-2). The objective of the experimentation phase-2 is to develop, compare, and evaluate the efficiency of the PAT. Fig. 6 shows the process of experimentation phase-2. In this experimentation, firstly, a source image is supplied to the PAT and TAT. The PAT is the composite transformation that combines translation, PIMRA, rotation, and shearing. To generate the fractal images, both functions (PAT & TAT) are executed on different types of images. The input images contain various types of image objects such as reflective regions, line structures, curved-shaped objects, symmetry objects, and foreground & background objects. After the execution of both PAT and TAT, the obtained results are compared to justify the performance of the PAT. In the generation of fractal images, it is noticed that the size of the image is highly reduced when the number of iterations is considerably increased. Hence, it is difficult to visualize the structural deformation of the image. To analyze the visual quality of the attractors that are generated in the last iteration a zoom function is utilized. In the OIQA, the value of the global SSIM of each fractal image is observed based on its local SSIM maps.

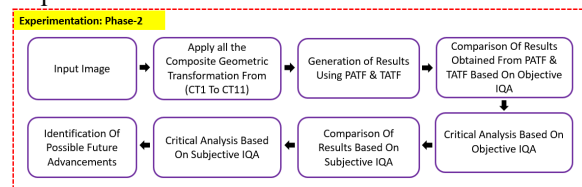


Fig. 6. Flow of Experimentation (Phase-2)

5. Dataset

To conduct the OIQA, the images are taken from two datasets, such as RetargetMe [45] and SIR² [46]. The RetargetMe dataset provides images which contains attributes such as line structure, curve-shaped objects, foreground and background objects, and symmetry objects. These attributes are important in comparison of PAT and TAT. The second dataset, i.e., the SIR² image dataset, is selected to obtain the images in which

reflective regions are present.

6. Results and Discussion

In the experimentation phase-1, firstly the existing and proposed geometric transformation techniques are compared, such as translation, rotation, and shearing. The obtained results from PAT and TAT showcase the similar visual quality of transformed images. The different values of SSIM obtained from these transformations are not presented in the paper as they are similar. Secondly, the results obtained from PIMRA and the scaling technique are compared by different values of chosen parameters. The IQA based on the results obtained from existing scaling and PIMRA is shown in Fig. 7. The figure showcases that for image type-1, the obtained global SSIM values from the PIMRA and scaling operator are 0.98 and 0.93, respectively. Therefore, the PIMRA produces less deformation on the curved-shaped objects present in the image. Similarly, for image type-2, the obtained global SSIM values from the PIMRA and scaling operator are 0.85 and 0.82, respectively. From the obtained values, it is noticed that the PIMRA and scaling operator both produce high structural deformations on the prominent line structures.

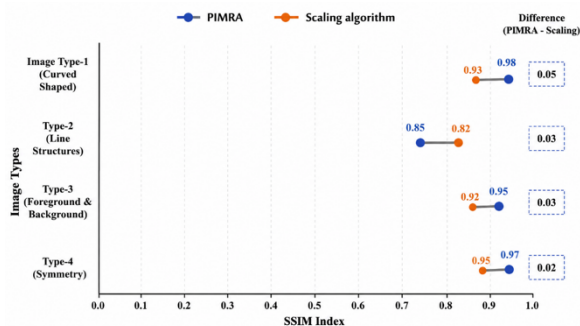
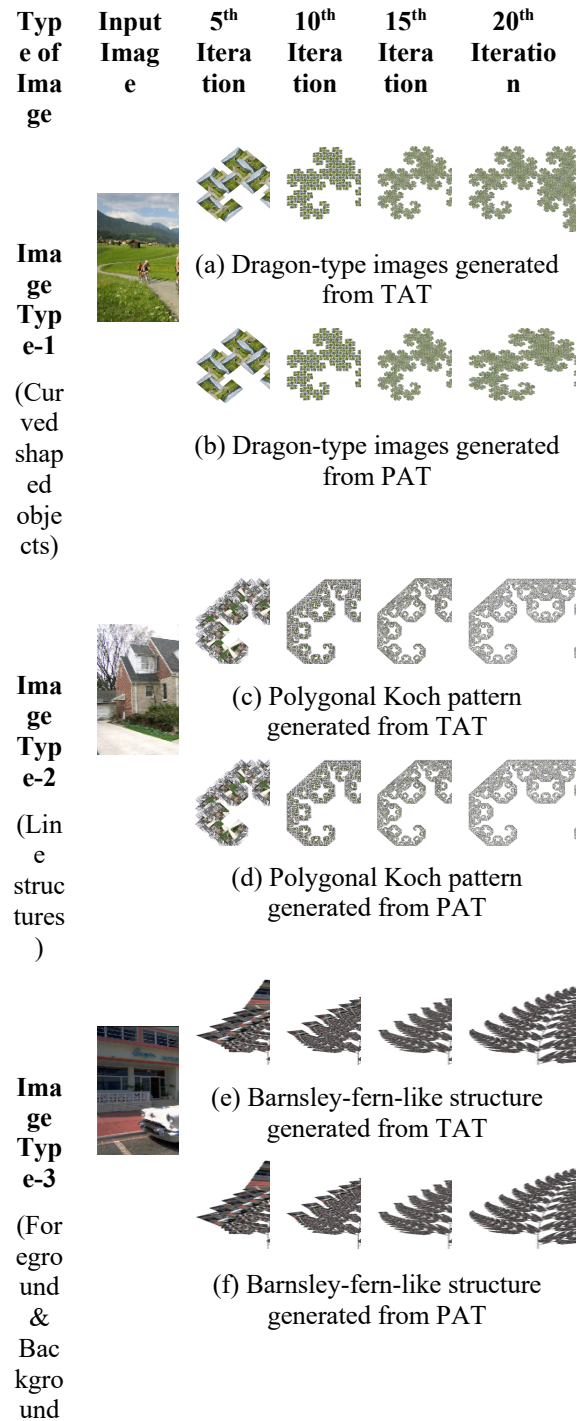


Fig. 7. OIQA based on SSIM

The obtained global SSIM values from the PIMRA are close to 1 as compared to the scaling operator, which signifies the efficiency of the PIMRA. Further, for image type-3, the obtained global SSIM values from the PIMRA and scaling operator are 0.95 and 0.92, respectively. In this case, the higher SSIM value justifies the performance of PIMRA when the image contains foreground and background objects. Finally, for image type-4, the obtained global SSIM values from the PIMRA and scaling operator are 0.97 and 0.95, respectively. Therefore, the PIMRA again produces less deformation on the image that contains symmetry objects. After computing the average percentage similarity technique-wise, it is found that the efficiency of PIMRA is 3.25% higher than that of the scaling operator. The average percentage similarity of PIMRA can be enhanced by increasing the number of iterations while transforming the input image. In experiment phase-2, all the proposed transformation

techniques along with PIMRA are combined and tested based on SSIM.

In experiment phase-2, a composite affine transformation is performed on different sets of images. The results are tested on different number of iteration to observe and justify the performance of PAT. Fig. (8-9) present fractal images constructed using PAT. The obtained results after 20th iteration are closely observed using a zoom function.



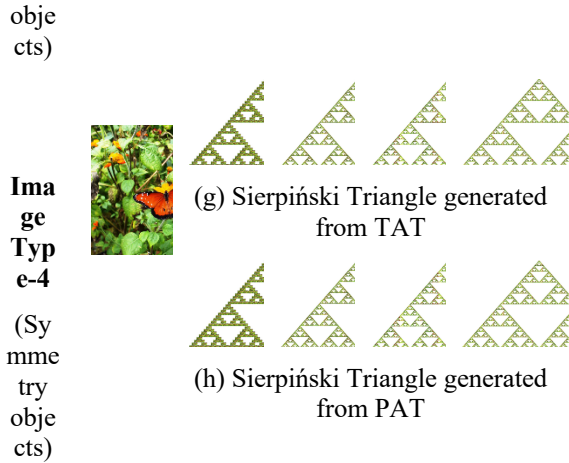


Fig. 8. Generation of fractal images based on different image types taken from the RetargetMe dataset

In Fig. 9, a total of four different types of images are selected from the SIR² dataset. The images are selected such that reflective regions are present at different spatial locations and under different scene conditions, including reflections in the central region, reflections in the foreground and background regions, and reflections on glossy surfaces. The variation in the spatial locations of the reflective regions helps assess whether PAT can preserve reflective regions along with the object boundaries that separate foreground and background regions where the color composition of both regions is almost similar.

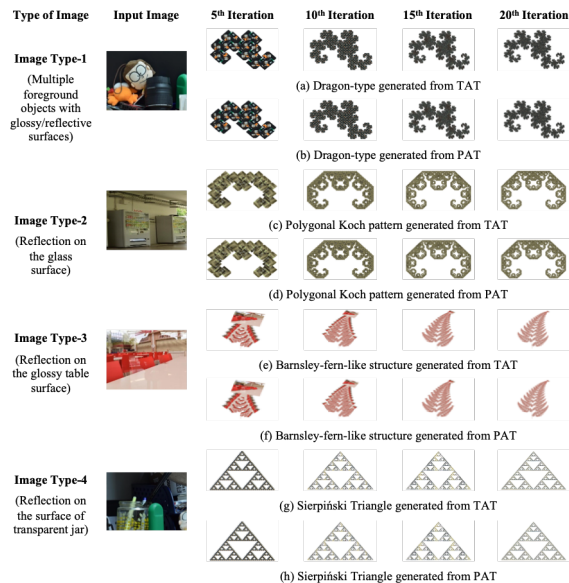
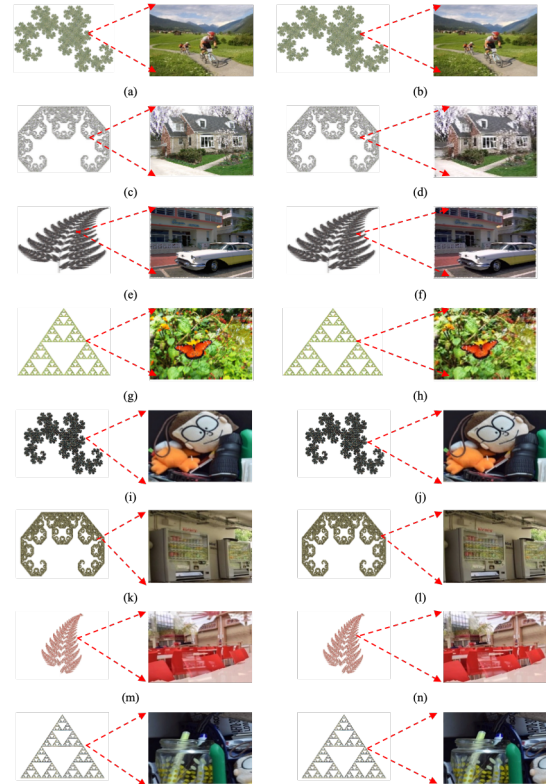


Fig. 9. Generation of fractal images based on different image types taken from the SIR² dataset

To visualize the smallest region of the fractal image obtained from PAT and TAT, a zoom operation is applied to all the image which are obtained after the

20th iteration. The results of the zoom operation are presented in Fig. 10.



The subsequent subsections present the assessment of the visual quality of zoomed images based on the amount of distortion. Therefore, the OIQA percentage similarity analysis and average percentage similarity analysis are carried out on the obtained results. To carry out similarity analysis, different values of parameters are taken into consideration.

➤ Percentage similarity analysis

In this section, the percentage similarity analysis is carried out on different types of images obtained at the 20th iteration, as shown in Fig. 8-9. Firstly, from Fig. 11, it is evident that the PAT produces less similarity variation in image type-1 (Curved shaped objects) as compared to TAT. Therefore, it is concluded that the PAT produces a high percentage of similarity, i.e., 71.4%, as compared to TAT, i.e., 68.37%. Secondly, for image type-2 (Line structures), both PAT & TAT produce a high percentage of deformation, i.e., 47.3% and 27.12% respectively. The main reason behind the high percentage distortion is due to the presence of highly overlapped line structures. Although the percentage of distortion produced by PAT is high, it still generates promising results as compared to TAT. Thirdly, for image type-3 (Foreground & background) and type-4 (Symmetry), PAT produces a high percentage of similarity as compared to TAT. In the case of image type-3, the PAT produces a high percentage of similarity, i.e., 53.7%, which is higher

than TAT, i.e., 41.71%. In the case of image type-4, the PAT produces a high percentage of similarity, i.e., 57.74%, which is higher than TAT, i.e., 51.17%. The percentage similarity analysis it is disclosed that the aforementioned attributes of images are highly responsible for degrading the performance of the scaling operator present in TAT. The PIMRA activates its three major operations, i.e., energy augmentation on the object boundaries, morphological operation that support seam diversion operation to suppress distortion in the salient objects present in the image. The performance of PAT is also test on 100 input images with other attributes, and OIQA is also carried out to test the performance of PAT. The results are not presented in this paper due to space constraints.

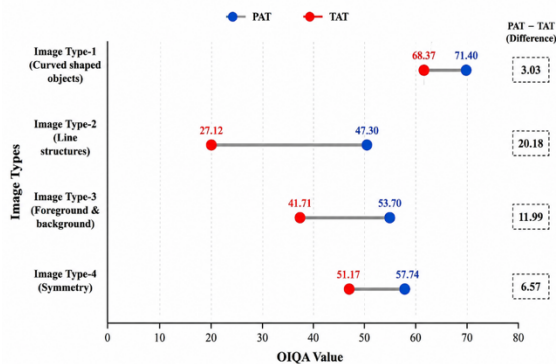


Fig. 11. Percentage similarity analysis based on images taken from RetargetMe Dataset

Further, the results obtained from the images selected from SIR² dataset are considered for OIQA and are shown in Fig. 12. The percentage similarity analysis is again carried out on different types of images obtained at 20th iteration as shown in Fig. 9. Firstly, from Fig. 12, it is evident that the PAT produces less similarity variation in image type-1 (Multiple foreground objects with glossy/reflective surfaces) as compared to TAT. Therefore, it is concluded that the PAT produces a high percentage of similarity, i.e., 81.4% as compared to TAT, i.e., 70.37%. Secondly, for image type-2 (Reflection on the glass surface), both PAT & TAT produce a high percentage of deformation, i.e., 57.3% and 46.12%, respectively. The main reason behind the high percentage distortion is due to the presence of highly overlapped objects. Thirdly, for image type-3 (Reflection on the glossy table surface) and type-4 (Reflection on the surface of transparent jar), PAT produces a high percentage of similarity as compared to the TAT. In the case of image type-3, the PAT produces a high percentage of similarity, i.e., 73.7%, which is higher than TAT, i.e., 52.71%. In the case of image type-4, the PAT produces a high percentage of similarity, i.e., 85.74%, which is higher than TAT, i.e., 79.17%. In the percentage similarity analysis, it is

disclosed that the performance of PAT is higher than TAT.

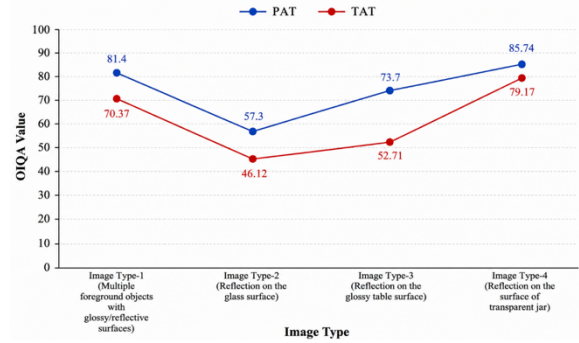


Fig. 12. Percentage similarity analysis based on images taken from SIR² Dataset

➤ Average percentage similarity analysis

Fig. 13 showcases the average similarity produced by PAT and TAT on different types of images. From the images selected from the RetargetMe dataset, the average percentage similarity obtained from PAT is 57.54%, which is 10.45% higher than that of TAT. Similarly, from the images selected from the SIR² dataset, the average percentage similarity obtained from PAT is 74.54%, which is 12.45% higher than TAT. The variations in the similarity values justify the performance of the PAT over TAT.

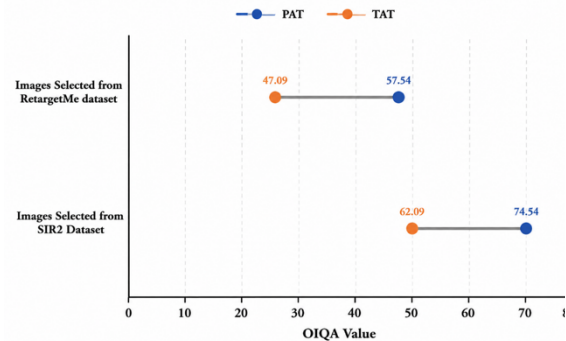


Fig. 13. Computation of technique wise average percentage similarity based on PAT and TAT

7. Conclusion & Future Scope

In fractal image generation using an iterated function system, affine transformations (translation, scaling, rotation, and shearing) are used to create patterns. However, scaling introduces noticeable distortions when changing the image aspect ratio. Experimental analysis shows that other transformations perform well, but scaling degrades image quality. To overcome this, the proposed approach combines geometric transformations with PIMRA to develop PAT, aiming to reduce distortion during downsampling. Performance is evaluated using parameters like luminance, contrast, and structure. Results show that translation, rotation, and shearing preserve image quality, while PIMRA produces significantly less distortion compared to scaling. Further evaluation

using OIQA reveals that images with strong line structures are most affected by deformation, though PIMRA helps reduce these effects and preserves properties like parallelism, collinearity, and symmetry. Overall, PAT generates better-quality fractal images with fewer distortions, especially for curved shapes and structured content. The PIMRA is also tested on images in which reflective regions are present. From the obtained values of OIQA, it is also evident that the performance of PAT in conjunction with PIMRA and other proposed geometric transformation techniques is higher than TAT. The experiments and analysis conducted in this paper conclude that PAT improves the visual quality of fractal images and reduces artifacts in the salient regions of the image. In the future, advanced energy maps can be integrated with PIMRA to improve its performance, and recent advanced techniques such as AI and ML can be integrated to automate seam selection and improve image retargeting efficiency.

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